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GENERAL DESIGN THEORY AND GENETIC EPISTEMOLOGY

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1 Introduction

General Design Theory is a domain independent theory of design proposed by Yoshikawa in which design and design knowledge are mathematically represented by using topology. Piaget's Genetic Epistemology is a theory about cognitive developments of children and has had great influences on developmental psychology in the last century. Genetic Epistemology presents mathematical structures for representing cognitive systems of human beings. We shall compare General Design Theory and Genetic Epistemology in order to show their tight connection. In particular, we discuss the differences of mathematical structures in these two theories, and we shall show expansions of General Design Theory and Genetic Epistemology from this comparison.

2 Design and Development

Developmental psychology is one of areas of psychology about children's developmental processes or transitions of mental phenomena like sensation, cognition and emotion [9, 3]. Genetic Epistemology (GE) is a theory about developments of logical thinking established by Piaget in 1920's [6, 2]. There are three fundamental ideas in GE: logical thinking can be regarded as logical operations, the cognitive structure of logical intelligence is domain independent, and developmental processes can be explained as a sort of biological adaptations by a notion of equilibration. By considering logical thinking as logical operations, Piaget studied the cognitive structures constructed by logical operations on cognitive objects, and he presented several mathematical structures as models of the cognitive structures. Although GE has been criticized in many aspects, GE is one of the most influential theories in developmental psychology in 20th century.

Developmental psychology has connections with design theory in the following two points. First, a design process is a creative and logical cognitive process which includes not only mere combinations of previous knowledge but also developments of logical thinking and knowledge which are discussed in developmental psychology. Then, knowledge in design includes various cognitive aspects and their acquisitions are argued in developmental psychology. Finally the structure of knowledge or logical thinking discussed in Piaget's GE corresponds to the topological spaces discussed in Yoshikawa's General Design Theory (GDT).

Our purpose is, by introducing the notion of hierarchy of knowledge or logical thinking, to show the common and different features of GDT and GE by comparing topological spaces in GDT and the mathematical structures in GE. Furthermore, from this comparison, we show that the distinction between "the ideal knowledge" and "the real knowledge" in GDT means the difference between two aspects of design knowledge simultaneously used in a design.

Designers can get these two sorts of knowledge through their developmental process of logical thinking, and we name these two aspects of knowledge as the *entity-oriented knowledge* and the *concept-oriented knowledge*. A theory with which we can deal with these two aspects of knowledge would be also necessary for theoretical investigations of design, and constructions of such a theory lead to an expansion of GDT and GE. Interactions of GDT and GE will provide new developments in design theory and developmental psychology.

3 General Design Theory

Yoshikawa's General Design Theory (GDT) [10] is an axiomatic theory of design without domain specification. In an axiomatic theory, definite vocabularies about fundamental concepts are given, and assertions about the vocabularies are called as propositions. Propositions are divided into the two categories: one is the most fundamental and irreducible proposition which is called an *axiom*, and the other is a proposition which can be deduced from axioms and it is called a *theorem*. The basic vocabularies in GDT include "entities", "entity concepts", and "abstract concepts", and abstract concepts include "functional concepts" and "attributed concepts". The entity set ***S*** is a set "which includes all entities in it as elements", entity concept set ***S'*** is a set of concepts "which have been formed according to the actual experience of an entity". The set of abstract concepts is "derived by the classification of concepts of entity according to the meaning or the value of entity". The following three axioms are presented in GDT as the basic properties of these concepts.

Axiom of Recognition: Any entity can be recognized or described by the attributes.

Axiom of Correspondence: The entity set ***S'*** and the set of concept of entity ***S*** have one-to-one correspondence.

Axiom of Operation: The set of abstract concept is a topology of the set of entity concept.

In GDT, there are two kinds of knowledge for design. A kind of perfect knowledge is named "the ideal knowledge" and more actual sort of knowledge named "the real knowledge". Above three axioms are valid only on the ideal knowledge. The ideal knowledge is represented by two topological spaces, the function space and the attribute space, and a map between the two topological spaces. Design is basically an activity which connects design specification and design solution. In GDT, design specification is denoted by functional concept set and design solution is described by concept of attribute set. In the real knowledge, design is an activity "to designate a domain on the attribute space which is included by that on the abstract concept set" and it can be discussed in terms of the continuity of a map between the function space and the attribute space. On the other hand, in the real knowledge, "the design is possible if and only if there is any rule of direct correspondence between the topologies of abstract concepts and of attribute concepts without intervention of the entity concepts".

It is proved in GDT that "design solution is obtained immediately after the specification is described" without design process in the ideal knowledge. On the other hand, in the real knowledge, the axioms of GDT do not work and the framework of GDT is invalid to discuss design. That is, design is not necessary under the ideal knowledge, and the axioms of GDT do not work under the real knowledge. Kikuchi and Nagasaka [4] called this situation as "the Paradox of General Design Theory", and showed this paradox is caused by the formulation of entity concepts in GDT. As mentioned above, the outline of design in GDT is valid only with the ideal knowledge, and we need a direct connection between functional concepts and attribute concepts in the real knowledge for design. Tomiyama and Yoshikawa [10] described this sort of the real knowledge "the real knowledge in a wide sense", they introduce a subset S^* of S' on which the axioms of GDT hold for representing the real knowledge. The real knowledge in this meaning is called "the real knowledge in a narrow sense" by them.

4 Genetic Epistemology

Piaget's Genetic Epistemology (GE) [6] is a comprehensive theory on developmental processes of logical thinking covering from the birth to the adolescence, and which has tremendous impacts on modern developmental psychology. In GE, logical thinking is regarded as a combination of cognitive operations on concrete and abstract objects. Piaget asserted about developmental processes of logical thinking that there are general cognitive structures in developmental processes and they can be explained by the concept of an equilibration which are caused by assimilations and accommodations and which is a kind of biological adaptation. The theory asserts also that developmental processes of logical thinking have four major stages, which are called "sensori-motor stage", "pre-operational stage", "concrete operational stage" and "formal operational stage". Piaget thought logical thinking as logical operations, and he discussed cognitive structures as the relationships between cognitive subjects and cognitive objects of logical operations, and he represented the structure of the relationship at each developmental stage by mathematical structures, like groups and lattices.

Babies at "the sensori-motor stage (0-2 years old)" acquire symbols and cognitive representations through reflex behaviours, repeated behaviours, and so on. At the succeeded "pre-operational stage (2-7 years old)", infants get to understand the nature and the relation of sensual objects according to the subjective attributes of egocentricity. The cognitive structures between cognitive subjects and objects at these two early stages are called *schema*. These schemas are thought to change in coordination with each other. Logical intellect at these two stages suggests the origin of logical operations at the later stages but does not include logical thinking with realistic physical activity because of inability of reversibility.

After transition to "the concrete operational stage (7-11years old)", the sensori-motor schemas becomes sensual cognitive structures with logical operations. When children come to think logically, they acquire reversibility and compensation in logical operation, and their cognitive objects come to include logical thinking such as classification and seriation. Piaget describes these concrete operations by mathematical structures named groupings which have 5 characteristics: combinativity, reversibility, associativity, general operation of identity and tautology. However, these logical operations depend on cognition of the peculiarity of concrete knowledge of objects. At next stage, children' logical operations come to be independent of cognitive objects.

The stage in which children come to think formally with logical and physical law, independent of concrete knowledge of objects, is called "the formal operational stage (11-15years old)". At this stage, cognitive objects of formal operations are logical forms, and logical thinking is beyond cognition of concrete attitude of objects by inevitable formal knowledge. The characteristics of this formal operation are represented by mathematical structures such as lattices and the INRC group based on Klein's four-group.

GE is verified and criticized by many researches until now. Particularly, many experimental evidences have been found that developments of logical thinking depend on domain specificity. The mathematical structures in GE are thought to be the framework of domain generality [3].

5 Mathematical Structure and Hierarchy of Knowledge

GDT has mathematical frameworks for formal representation of design knowledge in which topological spaces are used, and GE has also mathematical frameworks for formal representation of cognitive structures of logical thinking in which groups and lattices are used. All of these structures can be defined within set theory. A mathematical structure $\boldsymbol{M}$ is defined on a set $\boldsymbol{X}$ of the cognitive objects with operations or relations R and it is described as $\boldsymbol{M} = \langle \boldsymbol{X}, \boldsymbol{R} \rangle$. Topological spaces and groups are both mathematical structures. A topological space is a mathematical structure $\langle \boldsymbol{X}, \boldsymbol{O} \rangle$ where $\boldsymbol{X}$ is a set and $\boldsymbol{O}$ is a set of subsets of $\boldsymbol{X}$ with suitable conditions. A topological space is a generalization of a metric space and an Euclidean space. A group is a structure $\boldsymbol{G}=\langle \boldsymbol{X}, \{*, \boldsymbol{e}\} \rangle$ where $\boldsymbol{X}$ is a set, $*$ is a binary operation, and $\boldsymbol{e}$ is an identity element in $\boldsymbol{X}$. The INRC group in GE is a mathematical structure, after Klein's four-group, defined on a set {I, N, R, C} and each element means one of logical operations.

Both in GDT and GE, the operations on cognitive objects and knowledge are the main topics of the discussion. The knowledge about entity, entity concept and the entity are identified and correspondent by the Axiom of Correspondence. The abstraction of the knowledge of entity, the abstract concept set suggest logical operations of knowledge as set operations defined on the abstract concept set by the Axiom of Operation. On the other hand, in GE, cognitive subjects are the main of discussion and discussed and mainly logical operations are argued. Knowledge is suggested as cognitive objects of logical operations, so that the concrete objects of a classification and the propositions as objects of logical operations are thought to be knowledge in GE.

Then, we introduce the concept of *rank* according to the level of abstraction of knowledge. First, knowledge about physical existent objects is called rank 0 knowledge, abstraction of rank 0 knowledge are called rank 1 knowledge, and abstraction of rank 1 knowledge are called rank 2 knowledge. More formally, when a set $\boldsymbol{X}$ of rank objects 0 is given, the subsets of $\boldsymbol{X}$ is called rank 1 knowledge, and the set of rank n knowledge is defined inductively. For example, suppose $\boldsymbol{X}$ is a set of rank 0 knowledge, the open sets of the topological space defined on $\boldsymbol{X}$ are rank 1 knowledge, and set operations such as union and intersection are rank 2 knowledge. In the following, we call conveniently the mathematical structure constructed from rank n knowledge and rank n+1 knowledge as "the (n, n+1)-structure".

6 Comparison of DGT and GE

In GDT, the entity concept is thought to be rank 0 knowledge since entities and entity concepts are corresponding by Axiom of Correspondence, and an abstract concept is rank 1 knowledge. Axiom of Operation means that an entity concept set and an abstract concept set form a topological space as a (0, 1)-structure. Set operations on open sets of a topological space like union and intersection are operations on the rank 1 knowledge, so they can be thought as rank 2 knowledge. The map on the real knowledge is also thought to be rank 2 knowledge. However, these two kinds of rank 2 knowledge are referred intuitively and they are not treated formally in GDT.

In GE, the cognitive representations in the sensori-motor stage and defective operations in the pre-operational stage are rank 1 knowledge since they are operations on rank 0 knowledge. The schemas in these stages connect rank 0 knowledge which the rank 1 knowledge, so they are (0, 1)-structures. The combination and the reversibility of logical operations on rank 0 knowledge in the concrete operational stage are discussed in GE. These combination and reversibility of operations are defined on rank 1 knowledge, hence these operations are rank 2 knowledge and grouping is a (1, 2)-structure. In the formal operational stage, formal operations are defined on the rank 1 knowledge, therefore they are thought as rank 2 knowledge and INRC group is a (2, 3)-structure. The difference between the concrete operational stage and the formal operational stage is not only the axioms of grouping, group and lattice, but also the ranks of knowledge for logical operations.

Yoshikawa has noted that schemas in GE correspond to an open set of a topological space. Since an open set of abstract concepts is rank 1 knowledge and a schema is a (0, 1)-structure, a schema does not correspond to an open set directly and it should be compared with a topological space. However, when $\boldsymbol{X}$ is a subset of a set $\boldsymbol{S}$, $\boldsymbol{O_X}=\{\phi, \boldsymbol{X}, \boldsymbol{S}\}$ is a topology on $\boldsymbol{S}$ and $\langle \boldsymbol{S}, \boldsymbol{O_X} \rangle$ is a topological space. This topological space corresponds to schema and an open set $\boldsymbol{X}$ corresponds to a schema through the topological space $\boldsymbol{O_X}$. Yoshikawa's idea can be justified by this correspondence. The correspondence between knowledge in GE and GDT established by rank of cognitive knowledge can be thought as a generalization of Yoshikawa's idea.

A schema in GE directly corresponds to a topological space in GDT. However, the logical operations of intersection and union defined on $\boldsymbol{O_X}$ are discussed as concept operations in GDT. This structure defined on $\boldsymbol{O_X}$ is a (1, 2)-structure and it corresponds to the grouping in GE. We remark that all of the knowledge ranked 0, 1 and 2 are referred essentially in both discussions related in GDT and GE, and the difference between a topological space in GDT and a grouping in GE is the difference between "entity-oriented" and "concept-oriented" attitudes in partial mathematical descriptions of the relations among the three sorts of cognitive objects. That is, a topological space is a (0, 1)-structure and rank 0 knowledge is entity-oriented, and a grouping is a (1, 2)-structure and rank 2 knowledge is concept-oriented.

It is noted in GDT that the design with the real knowledge in a wide sense is represented by "direct correspondence between the topologies of abstract concepts and the topology of attribute concept without intervention of entity concepts", but, the direct correspondence among abstract concepts is rank 2 knowledge and the design knowledge with the real knowledge in a wide sense is represented by (1, 2)-structure, because abstract concepts is rank 1 knowledge. This structure corresponds to grouping. Knowledge at the formal operational stage logically connects formal objects like logic and physical laws, and this is just the real knowledge in a wide sense in GDT. The INRC group and the lattice at this stage are (2, 3)-structures and indirectly correspond to the structure for the real knowledge in a wide sense, and there is no structure in GDT which corresponds directly to INRC group.

Besides the above comparison between GDT and GE, there are some other obvious differences between GDT and GE. First, relations between two topological spaces, a functional space and an attribute space, is discussed in GDT, but relations of two mathematical structures is only suggested by the word "coordination" in GE. Next, constructions of topological spaces are not argued in GDT, but the method to composition of mathematical structures is discussed in GE.

Newton and Leibniz are famous as the originators of calculus. Although their systems are equivalent, it is well known that their basic ideas are quite different. Cassirer [1] compares their ideas about the nature of time and space, and he showed that the truth for Newton is so realistic that they should be founded in the objective nature or structure in the world, while the truth for Leibniz is so idealistic that the research of the objective nature or structure is nothing but to know the nature or cognitive structure of our mind. That is, their ontological standpoint is quite different, but Cassirer remarked that they have a common feature that they fight against the thesis of the British empiricism and that they are completely identified in the opinion that the structure of the world cannot be investigated by mere sensuous terms. We remark that both GDT and GE refuse sensuous or empirical research of cognitive activity although their basic attitude is so different: GDT is object-oriented and GE is concept-oriented. In this sense, there is a parallel relation between Newton and Leibniz in Cassirer's argument and Yoshikawa and Piaget in ours.

7 Expansion of GDT and GE

The ideal knowledge, the real knowledge in a wide sense and the real knowledge in a narrow sense in GDT can be thought as the three distinct fundamental standpoints for representation of design knowledge. The ideal knowledge and the real knowledge in a narrow sense are both expressed by a (0, 1)-structure and this corresponds to the schema at the sensori-motor stage and the pre-operational space as stated above. The real knowledge in a wide sense is expressed by a (1, 2)-structure, and this corresponds to a grouping in GE.

Since we can say that (0, 1)-structures are entity-oriented and (1, 2)-structures are concept-oriented, the ideal knowledge and the real knowledge in a narrow sense can be thought as entity-oriented knowledge, and the real knowledge in a wide sense can be considered as concept-oriented knowledge. The ideal knowledge and the real knowledge in GDT should not be thought as two sorts of exclusive knowledge, but they can be thought as the entity-oriented knowledge and the concept-oriented knowledge which can exist simultaneously in a design process. To discuss a schema and a grouping simultaneously, and to discuss a design with entity-oriented and real-oriented knowledge are equal to discuss a (0, 1)-structure and a (1, 2)-structure at the same time. In GE, there are many arguments about a (0, 1)-structure but insufficient about a (1, 2)-structure. GDT and GE are expected to complementally theories.

The fundamental problems in developmental psychology are whether a developmental process is continuous or discontinuous, and whether development is from nature or by nurture [7, 3]. If one supposes equilibration as the developmental principle like GE, developments should be continuous, but the idea of developmental stages means the existence of discontinuity, so these are superficial inconsistency in GE. But Piaget [6] thought the transitions between four stages of development process are kinds of asymptotic transitions and the beginning of the stages of logical thinking is hardly watched in "nurture". As Sternberg and Okazaki [9] mentioned, we should consider that these two properties exist at the same time as two aspects of a common developmental process, and we need a theory to explain both aspects without contradiction. The expansion of GDT including both of the entity-oriented knowledge and the concept-oriented knowledge corresponds to an expansion of GE with which continuity-or-discontinuity problem will be resolved.

8 Conclusion

In this paper, we discuss common and different natures of topological spaces in GDT and mathematical structures in GE by introducing the concept of hierarchy of objects, and we showed also that the difference of the two theories is not the difference of mathematical structures, but of the fundamental stances: GDT is entity-oriented, and GE is concept-oriented. From the result of the comparison, we showed a new viewpoint that the distinction between the ideal knowledge or the real knowledge in a narrow sense and the one in a wide sense is not a standpoint for mathematical formulations of design knowledge but the distinction between two sorts of design knowledge that exist simultaneously in a design process. The expansion of GDT to a theory including these two kinds of design knowledge is anticipated to be correspondent to that of GE in which the fundamental problem about continuity and discontinuity in a development process can be solved. We can expect that exchanges between GDT and GE present new expansion in each area.

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